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Electric and weak electric dipole form factors for heavy fermions in a general two Higgs doublet model

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Abstract. The electric and weak electric dipole form factors for heavy fermions are calculated in the context of the most general two Higgs doublet model (2HDM). We find that a large top mass can produce a significant enhancement of the electric dipole form factor in the case of the b and c quarks. This effect can be used to distinguish between different 2HDM scenarios.

1 Introduction

One of the simplest extensions of the standard electroweak model (SM) is the so-called two Higgs doublet model (2HDM), in which the new ingredient is the presence of a second doublet of scalar fields. The inclusion of these new fields implies various phenomenological consequences, which have led the 2HDM to be subject of analysis during the last two decades.

We concentrate here on one of the most interesting features concerning the 2HDM, which is the presence of different sources of CP violation beyond the standard δ_{CKM} phase in the quark-mixing matrix. In particular, we analyze the effects of the new parameters of the model on the CP-violating electric and weak electric dipole form factors for heavy fermions. The interest in these observables has increased in recent years, in view of the ongoing activity both in the theoretical and in the experimental areas [1–3]. SM predictions for CP-odd dipole moments are extremely small, and this opens the possibility for one-loop effects coming from extended models to show up [4, 5]. Specific observables have been proposed and studied in the literature [1,2,6], and some bounds have already been obtained from experimental measurements in e^+e^- collisions [3,7].

If done in a completely general way, the addition of a second scalar doublet to the SM Lagrangian is problematic: one immediately finds that a general 2HDM model contains tree-level flavor-changing neutral currents (FCNC), which are strongly suppressed phenomenologically. To avoid this problem, it is usual to introduce ad hoc discrete symmetries, in such a way that all fermions of a given charge couple to only one of the doublets [8]. This can be done in different ways, leading to the so-called 2HDM I and II. It is often said that the obtained flavor conservation is "natural". The inclusion of discrete symmetries, however, is not the only way of preventing the undesired FCNC [9–11]. In fact, the presence of strong hierarchies in the fermion masses and mixing angles seems to be a clear signature of an underlying theory of flavor beyond the SM Yukawa couplings. From this point of view, it can be also "natural" to expect that the suppression of FCNC observed at low energies could be explained in the context of this by now unknown theory. On the other hand, whereas the phenomenological constraints on FCNC are very stringent for processes which involve the first family of quarks and leptons, this is not the case if one considers only the mixing between the second and third fermion families. One possibility is to assume that the suppression of FCNC is related to the masses of the involved fermions, as has been proposed by several authors [10,11].

Here, instead of choosing a particular Ansatz to enforce the suppression of tree-level FCNC, we will consider a completely general 2HDM, using a convenient parametrization to take into account the existing phenomenological constraints. As has been pointed out recently [12], the various sources of CP violation can be classified into four classes:

(1) CP violation in charged and neutral flavor-conserving scalar exchange;
(2) CP violation in neutral flavor-changing scalar exchange;
(3) CP violation in the neutral scalar mixing matrix;
(4) CP violation in charged gauge boson exchange (the usual CKM mechanism).

It is clear that particular 2HDMs show in general different patterns for these CP violation sources. In this paper we analyze and compute the 2HDM predictions for the flavor-diagonal CP-odd couplings of heavy fermions, both quarks and leptons, to the neutral gauge bosons γ and Z. For on-shell fermions and gauge bosons, the corresponding $\bar{f}f\gamma$ and $\bar{f}fZ$ form factors are known as the electric

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dipole moment (EDM) and weak electric dipole moment (WEDM) of the fermion f, respectively. The presence of a nonvanishing dipole moment of this kind is a signal of time reversal symmetry violation, and in our framework, of CP violation. In general, the form factors are gauge invariant quantities – and can be contrasted with experiment – only when the external fermions and gauge bosons are on the mass shell. However, it can be seen that in the 2HDM the one-loop predictions for the electric and weak electric dipole form factors are still gauge invariant when the gauge bosons are off-shell.

We will present the analysis for the CP-violating dipole form factors in the general 2HDM case, and then apply the results to models which include discrete symmetries and models in which the magnitude of the FCNC is related to the masses of the fermions involved. As stated, the fermion electric and weak electric dipole form factors are nonzero only at high orders in the SM, whereas 2HDM give rise in general to nonvanishing contributions at the level of one loop. Therefore, they represent good candidates for an observation of CP-odd effects arising from an extended scalar sector. In the case of heavy fermions, the effects are particularly important, since the new (scalar-mediated) contributions are proportional to nonnegative powers of the fermion masses.

The paper is organized as follows: in Sect. 2, the notation and the general 2HDM considered in this work are presented. The analytical and numerical results for the CP-violating form factors are given in Sect. 3, while Sect. 4 contains our conclusions. In the Appendix we quote some explicit expressions for the Feynman integrals used in our analysis.

2 Model

As stated, we here consider a completely general 2HDM, allowing in principle for the presence of tree-level FCNC. We adopt in the following the notation introduced in [12], where the most general Higgs potential is parametrized as

$$\begin{aligned} V(\phi_1,\phi_2) = & -\mu_1^2\phi_1^\dagger\phi_1 - \mu_2^2\phi_2^\dagger\phi_2 - (\mu_{12}^2\phi_1^\dagger\phi_2 + \text{h.c.}) \\ & +\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2) \\ & +\lambda_4(\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right] \\ & + \left[(\lambda_6\phi_1^\dagger\phi_1 + \lambda_7\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{h.c.}\right], \end{aligned} \tag{1}$$

and the neutral scalars acquire vacuum expectation values

$$\langle\phi_1^0\rangle = \frac{v}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\delta}\cos\beta, \quad \langle\phi_2^0\rangle = \frac{v}{\sqrt{2}}\sin\beta. \tag{2}$$

In order to write the scalar–fermion couplings, it is convenient to introduce a new basis for the scalars, namely

$$\begin{aligned} \phi_1 &= \mathrm{e}^{\mathrm{i}\delta}(\cos\beta\Phi_1 + \sin\beta\Phi_2), \\ \phi_2 &= \sin\beta\Phi_1 - \cos\beta\Phi_2 \end{aligned} \tag{3}$$

with

$$\begin{aligned} \Phi_1 &= \begin{pmatrix} G^+ \\ (v + H^0 + \mathrm{i}G^0)/\sqrt{2} \end{pmatrix}, \\ \Phi_2 &= \begin{pmatrix} H^+ \\ (R + \mathrm{i}I)/\sqrt{2} \end{pmatrix}. \end{aligned} \tag{4}$$

It is easy to see that $H^\pm$ are physical charged scalar particles, while $G^\pm$ and G^0 are the Goldstone bosons corresponding to the spontaneous gauge symmetry breakdown. The remaining neutral scalars H^0, R and I are not in general mass eigenstates. In terms of these fields, the scalar–fermion couplings can be written as

$$L_Y = -(\sqrt{2}G_F)^{1/2}(L^{(nt)} + \sqrt{2}L^{(ch)}), \tag{5}$$

with

$$\begin{aligned} L^{(nt)} = & (\bar{U}_L M^u U_R + \bar{D}_L M^d D_R + \bar{L}_L M^l L_R)H^0 \\ & +(\bar{D}_L \Gamma^d D_R + \bar{L}_L \Gamma^l L_R)(R + \mathrm{i}I) \\ & +\bar{U}_L \Gamma^u U_R(R - \mathrm{i}I) + \text{h.c.}, \\ L^{(ch)} = & \bar{U}_L V_{\text{CKM}} \Gamma^d D_R H^+ - \bar{D}_L V_{\text{CKM}}^\dagger \Gamma^u U_R H^- \\ & +\bar{N}_L V_l \Gamma^l L_R H^+ + \text{h.c.}, \end{aligned} \tag{6}$$

where we have used the definitions $U = (u,c,t)^{\mathrm{T}}$, $D = (d,s,b)^{\mathrm{T}}$, $L = (e,\mu,\tau)^{\mathrm{T}}$ and $N = (\nu_e,\nu_\mu,\nu_\tau)^{\mathrm{T}}$. The couplings of the Goldstone bosons G^0 and $G^\pm$ are the standard ones. As we have checked that they do not contribute to the form factors we are interested in, we have not written them explicitly in the Lagrangian (6). As usual, the M^f with $f = u,d,l$ stand for the quark and lepton diagonal mass matrices and V_{CKM} is the Cabibbo–Kobayashi–Maskawa matrix (the subindex CKM will be omitted in the following to simplify the notation), whereas Γ^f, $f = u,d,l$ are arbitrary 3×3 complex matrices that arise from the extended Yukawa couplings. It is useful to distinguish between the diagonal and nondiagonal elements of Γ^f, defining [12]

$$(\Gamma^f)_{ij} = \begin{cases} \xi_{f_i} m_{f_i},\ i = j \\ \mu_{ij}^f,\ i \neq j \end{cases}, \tag{7}$$

where ξ_{f_i} and μ_{ij}^f are in general complex numbers. The parameters μ_{ij}^f are responsible for the tree-level flavor-changing neutral currents.

Nonzero phases in ξ_{f_i} and μ_{ij}^f represent new sources of CP violation beyond the standard δ_{CKM} phase in V. In addition, a further source of CP violation arises from the neutral scalar mixing: in the limit where CP is conserved, the CP-even states H^0 and R do not mix with I, which is CP-odd; however, the nonhermitian terms in the Higgs potential can induce either explicit or spontaneous CP violation [the latter arises from the phase δ in (2)]. Then, in general, one expects the neutral scalars to become mixed. The physical neutral mass eigenstates H_i^0 $(i = 1,2,3)$ can be written as

$$H_i^0 = \sum_{S=H^0,R,I} O_{Si} S, \tag{8}$$

O being an orthogonal (real) matrix. It is clear that in general the H_i^0 are not eigenstates of CP.

With the introduction of discrete symmetries to prevent FCNC, the above general structure becomes simplified. By requiring the Yukawa couplings to be invariant under the changes

$$\begin{aligned} \phi_1 &\to -\phi_1, \quad \phi_2 \to \phi_2, \\ D_{Ri} &\to -D_{Ri}, \quad L_{Ri} \to -L_{Ri}, \end{aligned} \tag{9}$$

together with $U_{Ri} \to -U_{Ri}$ ($U_{Ri} \to U_{Ri}$), one obtains the so-called 2HDM I (II). The 3×3 Γ matrices in (6) are then given by

$$\Gamma^{d,l} = \tan\beta M^{d,l}, \quad \Gamma^u = \begin{cases} \tan\beta M^u & \text{(model I)} \\ -\cot\beta M^u & \text{(model II)} \end{cases}. \tag{10}$$

If the Higgs potential is also invariant under the transformations (9), all CP-violating terms in (1) turn out to be forbidden. However, one can allow for a soft breakdown of the discrete symmetry, retaining the CP violation only through the coupling with μ_{12}^2 in (1). The electric and weak electric dipole form factors for the top quark have been analyzed within this scheme in [4]. Notice that, in this case, the parameters in (7) satisfy $\mathrm{Im}\xi_{f_i} = \mu_{ij}^f = 0$, so that the only source of CP violation beyond the SM is the mixing of CP-even and CP-odd fields in (8).

As commented on above, some models imply specific relations between the magnitude of the FCNC and the masses of the involved fermions. One usual Ansatz is that proposed by Cheng and Sher [10], in which the matrix elements of $\Gamma^{u,d}$ are governed by the order of magnitude of the fermion masses, obeying

$$\Gamma_{ij}^{u,d} = \lambda_{ij}\sqrt{m_i m_j}, \tag{11}$$

with λ_{ij} not far from unity. In general, if the lightest scalar masses are assumed to be in the region of a few hundreds GeV, bounds from $\Delta F = 2$ processes ($F = S, C, B$) constrain the couplings λ_{sd}, λ_{sb} and λ_{uc} to be $\lesssim 0.1$ [13]. Nevertheless, the presently available experimental information does not provide such a kind of constraints for λ_{ct}, which in principle is allowed to be $\mathcal{O}(1)$. Phenomenological consequences of having large c–t flavor-changing couplings have been studied recently by several authors [14]. We show below that the assumption of the Cheng–Sher Ansatz of (11) with $\lambda_{ct} \sim 1$ leads to a significant enhancement in the electric dipole form factors of the c quark.

3 Analytical and numerical results for CP-violating dipole form factors

The most general Lorentz invariant matrix element for the interaction of a gauge boson B with two on-shell fermions f, $\bar f$ can be written as

$$\begin{aligned} &\langle f(p_-)\bar f(p_+)|J_B^\mu(0)|B(q)\rangle \\ &= \mathrm{i}\, e\bar u_f(p_-)\Big[F_V^{B,f}(q^2)\gamma_\mu + F_A^{B,f}(q^2)\gamma_\mu\gamma_5 \\ &\quad + \left(F_S^{B,f}(q^2) + F_{AN}^{B,f}(q^2)\gamma_5\right) q_\mu \\ &\quad + \left(F_M^{B,f}(q^2) + F_E^{B,f}(q^2)\gamma_5\right)\sigma_{\mu\nu}q^\nu\Big] v_f(p_+), \end{aligned} \tag{12}$$

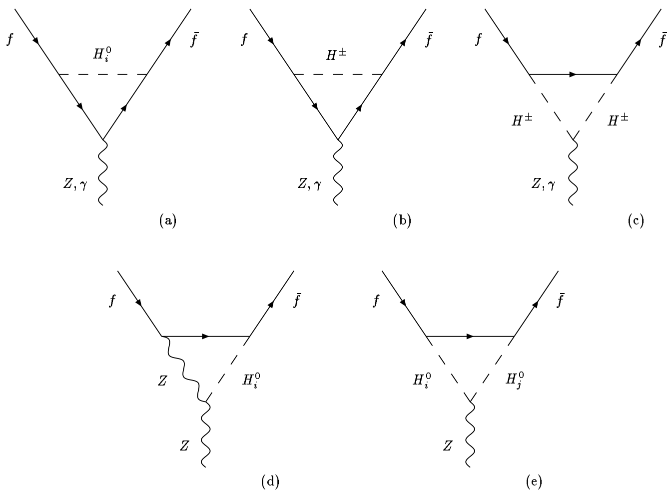

Fig. 1a–e. One-loop contributions to the form factors $F_E^Z(q^2)$ and $F_E^\gamma(q^2)$ in the 2HDM

where $q \equiv p_+ + p_-$, e is the proton charge and the coefficients $F_j^{B,f}$, so-called form factors, are in general functions of q^2. At the tree level, only the vector and axial vector form factors can be different from zero in a gauge theory.

The last two coefficients, $F_M^{B,f}$ and $F_E^{B,f}$, are known as magnetic (weak magnetic) and electric (weak electric) dipole form factors when one considers the coupling with the gauge boson $B = \gamma$ ($B = Z$). Both $F_M^{B,f}$ and $F_E^{B,f}$ are chirality-flipping quantities. Here we are interested in particular in $F_E^{B,f}$, which is CP-odd and vanishing small in the SM. We remark that within the SM all the above form factors $F_i^{B,f}(q^2)$ are gauge independent only when the gauge boson B is on-shell. In this case, $F_E^{B,f}$ is called the electric ($B = \gamma$) or weak electric ($B = Z$) dipole moment of the fermion f.

We will analyze the values of $F_E^{B,f}(q^2)$ ($B = \gamma, Z$) for heavy fermions in the 2HDM. Within these models one gets in general contributions already at the one-loop level, and the form factors are still gauge invariant when the gauge bosons are off-shell. Since these contributions are due to the exchange of neutral and charged Higgs bosons (which carry the CP-violation effects beyond the SM), and the Higgs–fermion couplings are proportional to the corresponding fermion masses, only heavy fermions are expected to yield significant effects. We will concentrate in particular on the electric and weak electric form factors for the τ lepton and the t, b and c quarks.

In the most general 2HDM, the relevant diagrams that contribute to $F_E^{B,f}$ at one loop are shown in Fig. 1 [notice that those in Fig. 1d and e only contribute to the $Zf\bar f$ form factor]. Let us begin by quoting the results for $F_E^{Z,f}(q^2)$, f being an up-like quark. The contributions from the diagrams of Fig. 1a–e are given by

$$
\begin{aligned}
\text{(a)}\quad F_E^{Z,f}(q^2) = {} & \frac{2\sqrt{2}G_F}{\sin\theta_{\rm W}\cos\theta_{\rm W}} g_V^u \\
& \times \sum_{f'=u,c,t} m_{f'} \sum_{j=1}^{3} \Big[m_f^2 O_{H^0 j}\delta_{ff'} \\
& \times (\mathrm{Im}\xi_f O_{Rj} - \mathrm{Re}\xi_f O_{Ij}) \\
& - \mathrm{Re}(\mathit{\Gamma}^u_{ff'}\mathit{\Gamma}^u_{f'f}) O_{Rj} O_{Ij} \\
& + \frac{1}{2}\mathrm{Im}(\mathit{\Gamma}^u_{ff'}\mathit{\Gamma}^u_{f'f}) \left((O_{Rj})^2 - (O_{Ij})^2\right) \Big] \\
& \times I^{(\mathrm{I})}(m_f, m_{f'}, m_{H^0_j}, q^2) \qquad \text{(13a)}
\end{aligned}
$$

$$
\begin{aligned}
\text{(b)}\quad F_E^{Z,f}(q^2) = {} & \frac{2\sqrt{2}G_F}{\sin\theta_{\rm W}\cos\theta_{\rm W}} g_V^d \\
& \times \sum_{f'=d,s,b} m_{f'} \mathrm{Im}\left[(\mathit{\Gamma}^{u\dagger} V)_{ff'} (V\mathit{\Gamma}^d)^*_{ff'} \right] \\
& \times I^{(\mathrm{I})}(m_f, m_{f'}, m_{H^\pm}, q^2) \qquad \text{(13b)}
\end{aligned}
$$

$$
\begin{aligned}
\text{(c)}\quad F_E^{Z,f}(q^2) = {} & \frac{2\sqrt{2}G_F}{\sin\theta_{\rm W}\cos\theta_{\rm W}} g^h \\
& \times \sum_{f'=d,s,b} m_{f'} \mathrm{Im}\left[(\mathit{\Gamma}^{u\dagger} V)_{ff'} (V\mathit{\Gamma}^d)^*_{ff'} \right] \\
& \times I^{(\mathrm{II})}(m_f, m_{f'}, m_{H^\pm}, q^2) \qquad \text{(13c)}
\end{aligned}
$$

$$
\begin{aligned}
\text{(d)}\quad F_E^{Z,f}(q^2) = {} & \frac{2\sqrt{2}G_F}{\sin\theta_{\rm W}\cos\theta_{\rm W}} g_V^u m_Z^2 m_f \\
& \times \sum_{j=1}^{3} (\mathrm{Im}\xi_f O_{Rj} - \mathrm{Re}\xi_f O_{Ij}) O_{H^0 j} \\
& \times I^{(\mathrm{III})}(m_f, m_{H^0_j}, q^2) \qquad \text{(13d)}
\end{aligned}
$$

$$
\begin{aligned}
\text{(e)}\quad F_E^{Z,f}(q^2) = {} & \frac{\sqrt{2}G_F}{2\sin\theta_{\rm W}\cos\theta_{\rm W}} m_f \\
& \times \sum_{f'=u,c,t} \left(|\mu_{ff'}|^2 - |\mu_{f'f}|^2 \right) \\
& \times \sum_{k<j} (O_{Rj}O_{Ik} - O_{Ij}O_{Rk}) \\
& \times (O_{Rj}O_{Rk} + O_{Ij}O_{Ik}) \\
& \times I^{(\mathrm{IV})}(m_f, m_{f'}, m_{H^0_j}, m_{H^0_k}, q^2) \qquad \text{(13e)}
\end{aligned}
$$

where

$$
g_V^u = (\frac{1}{2} - \frac{4}{3}\sin^2\theta_{\rm W}), \qquad g_V^d = (-\frac{1}{2} + \frac{2}{3}\sin^2\theta_{\rm W}),
$$
$$
g^h = (-\frac{1}{2} + \sin^2\theta_{\rm W}),
$$

and the Feynman integrals $I^{(\mathrm{i})}$ are quoted in Appendix A. The gauge invariance of the form factors has been explicitly checked. For down-like quarks and leptons, the resulting expressions are similar to those in (13). In the case of the down quarks these are obtained just by replacing

$$
\begin{aligned}
\mathit{\Gamma}^u, g_V^u &\longleftrightarrow \mathit{\Gamma}^d, g_V^d, \\
\sum_{f'=u,c,t} &\longleftrightarrow \sum_{f'=d,s,b} \\
O_{Ij} &\longrightarrow -O_{Ij}, \\
g^h &\longrightarrow -g^h, \\
V &\longrightarrow V^\dagger. \qquad (14)
\end{aligned}
$$

For the charged leptons, the diagrams in Fig. 1b and c are zero in the limit of vanishing neutrino masses. The remaining contributions can be obtained from those in (13) through the changes

$$
\begin{aligned}
g_V^u &\longrightarrow g_V^l, \\
\sum_{f'=u,c,t} &\longrightarrow \sum_{f'=e,\mu,\tau}, \\
O_{Ij} &\longrightarrow -O_{Ij}, \qquad (15)
\end{aligned}
$$

where $g_V^l = -1/2 + 2\sin^2\theta_{\rm W}$.

Finally, the contributions to the electric dipole form factors $F_E^{\gamma,f}$ are also easily obtained from the corresponding expressions for $F_E^{Z,f}$. In this case the rule simply consists in the replacements

$$
\frac{g_V^f}{\sin\theta_{\rm W}\cos\theta_{\rm W}} \longrightarrow 2Q_f, \qquad \frac{g^h}{\sin\theta_{\rm W}\cos\theta_{\rm W}} \longrightarrow 1 \qquad (16)
$$

for $f = u, d, l$. As stated, the diagrams in Fig. 1d and e do not contribute to $F_E^{\gamma,f}$.

Now we can make use of these results to evaluate the leading contributions to the electric and weak electric dipole form factors for the t, b and c quarks and the τ lepton. The final expressions can be written as

$$
\begin{aligned}
F_E^{Z,t}(q^2) &= \sum_{j=1}^{3} \left(a_j^{Z,t}\alpha_j^t + d_j^{Z,t}\gamma_j^t \right), \\
F_E^{\gamma,t}(q^2) &= \sum_{j=1}^{3} a_j^{\gamma,t}\alpha_j^t, \\
F_E^{Z,b}(q^2) &= (b+c)^{Z,b}\beta^b + \sum_{j=1}^{3} d_j^{Z,b}\gamma_j^b, \\
F_E^{\gamma,b}(q^2) &= (b+c)^{\gamma,b}\beta^b + \sum_{j=1}^{3} a_j^{\gamma,b}\alpha_j^b, \\
F_E^{Z,\tau}(q^2) &= \sum_{j=1}^{3} d_j^{Z,\tau}\gamma_j^\tau, \\
F_E^{\gamma,\tau}(q^2) &= \sum_{j=1}^{3} a_j^{\gamma,\tau}\alpha_j^\tau, \\
F_E^{Z,c}(q^2) &= \sum_{j=1}^{3} \left({a'_j}^{Z,c}{\alpha'_j}^c + d_j^{Z,c}\gamma_j^c \right), \\
F_E^{\gamma,c}(q^2) &= \sum_{j=1}^{3} \left(a_j^{\gamma,c}\alpha_j^c + {a'_j}^{\gamma,c}{\alpha'_j}^c \right) + (b'+c')^{\gamma,c}\beta'^c. \qquad (17)
\end{aligned}
$$

Table 1. Dominant coefficients for $F_E^{Z,t}(s)$ and $F_E^{\gamma,t}(s)$ for different values of s and the Higgs mass m_H. All values are in units of $e$cm

	$s^{1/2} = 500\,\text{GeV}$		$s^{1/2} = 1800\,\text{GeV}$	
	$m_H = 100\,\text{GeV}$	$m_H = 200\,\text{GeV}$	$m_H = 100\,\text{GeV}$	$m_H = 200\,\text{GeV}$
$a^{Z,t}$	$(-0.1 + 1.4\text{i}) \times 10^{-19}$	$(0.2 + 1.0\text{i}) \times 10^{-19}$	$(-1.2 + 1.0\text{i}) \times 10^{-20}$	$(-1.0 + 1.0\text{i}) \times 10^{-20}$
$d^{Z,t}$	$(1.0 - 2.8\text{i}) \times 10^{-20}$	$(0.8 - 3.5\text{i}) \times 10^{-20}$	$(3.1 - 2.6\text{i}) \times 10^{-21}$	$(3.0 - 2.6\text{i}) \times 10^{-21}$
$a^{\gamma,t}$	$(-0.3 + 4.1\text{i}) \times 10^{-19}$	$(0.6 + 2.9\text{i}) \times 10^{-19}$	$(-3.4 + 2.9\text{i}) \times 10^{-20}$	$(-2.9 + 2.7\text{i}) \times 10^{-20}$

Table 2. Dominant coefficients for $F_E^{Z,b}(s)$ and $F_E^{\gamma,b}(s)$ for $m_H = 100$ and $200\,\text{GeV}$ and different values of s. All values are in units of $e$cm

	$s^{1/2} = 10\,\text{GeV}$	$s^{1/2} = m_Z$	$s^{1/2} = 170\,\text{GeV}$	$s^{1/2} = 500\,\text{GeV}$
		$m_H = 100\,\text{GeV}$		
$(b+c)^{Z,b}$	5.0×10^{-21}	5.1×10^{-21}	5.7×10^{-21}	$(2.8 + 5.9\text{i}) \times 10^{-21}$
$d^{Z,b}$	3.6×10^{-21}	4.0×10^{-21}	6.3×10^{-21}	$(-1.1 + 1.8\text{i}) \times 10^{-21}$
$a^{\gamma,b}$	$(-0.8 - 0.2\text{i}) \times 10^{-22}$	$(-0.7 - 2.6\text{i}) \times 10^{-23}$	$(0.2 - 1.4\text{i}) \times 10^{-23}$	$(2.1 - 2.7\text{i}) \times 10^{-24}$
$(b+c)^{\gamma,b}$	1.0×10^{-20}	1.0×10^{-20}	1.1×10^{-20}	$(0.5 + 1.3\text{i}) \times 10^{-20}$
		$m_H = 200\,\text{GeV}$		
$(b+c)^{Z,b}$	3.0×10^{-21}	3.1×10^{-21}	3.3×10^{-21}	$(3.4 + 3.2\text{i}) \times 10^{-21}$
$d^{Z,b}$	2.0×10^{-21}	2.1×10^{-21}	2.4×10^{-21}	$(-0.5 + 2.5\text{i}) \times 10^{-21}$
$a^{\gamma,b}$	$(-2.3 - 0.4\text{i}) \times 10^{-23}$	$(-0.6 - 0.9\text{i}) \times 10^{-23}$	$(-0.2 - 0.7\text{i}) \times 10^{-23}$	$(0.8 - 2.2\text{i}) \times 10^{-24}$
$(b+c)^{\gamma,b}$	6.5×10^{-21}	6.6×10^{-21}	7.0×10^{-21}	$(6.5 + 7.8\text{i}) \times 10^{-21}$

In this parametrization, the dependence of $F_E^{B,f}$ on the unknown quark mass-matrix parameters and Higgs-mixing angles has been collected in the factors in Greek letters, whereas the coefficients $k_j^{B,f}$, with $k = a, a', \cdots, d$, contain the global factors including gauge boson couplings and fermion and Z masses, plus the Feynman integrals, which depend on q^2 and the masses of the Higgs bosons. The letters a, b, c, d identify the diagram from which each contribution originates, according to the notation in Fig. 1 and (13). We have distinguished with primes the contributions that include tree-level flavor-changing effects.

In (17) we have quoted only the dominant terms arising from the expressions (13); hence the contributions proportional to the light fermion masses have been neglected. In addition, the contributions from the diagram in Fig. 1e, which are proportional to the flavor-changing parameters, have been neglected in comparison to flavor-changing terms arising from the diagram in Fig. 1a [notice that (13e) vanishes when the matrices Γ^f are hermitian].

The explicit expressions for the factors in Greek letters can easily be obtained from (13). We find

$$\alpha_j^f = O_{H^0 j}\left[O_{Rj}\text{Im}\xi_f - \epsilon O_{Ij}\text{Re}\xi_f\right] - \epsilon\text{Re}(\xi_f^2) O_{Rj} O_{Ij} + \frac{1}{2}\text{Im}(\xi_f^2)\left(O_{Rj}^2 - O_{Ij}^2\right), \tag{18a}$$

$$\beta^b = -\text{Im}(\xi_t \xi_b), \tag{18b}$$

$$\gamma_j^f = O_{H^0 j}\left(O_{Rj}\text{Im}\xi_f - \epsilon O_{Ij}\text{Re}\xi_f\right), \tag{18c}$$

$$\alpha'^c = -\frac{\text{Re}(\mu_{ct}\mu_{tc})}{m_c m_t} O_{Rj} O_{Ij} + \frac{1}{2}\frac{\text{Im}(\mu_{ct}\mu_{tc})}{m_c m_t}(O_{Rj}^2 - O_{Ij}^2), \tag{18d}$$

$$\beta'^c = \sqrt{\frac{m_c}{m_t}}|V_{cb}|^2\text{Im}(\xi_c\xi_b) + \sqrt{\frac{m_s}{m_b}}\frac{\text{Im}(\mu_{tc}\mu_{sb}V_{cs}V_{tb}^*)}{\sqrt{m_s m_c m_b m_t}} + \frac{\text{Im}(\mu_{tc}\xi_b V_{cb} V_{tb}^*)}{\sqrt{m_c m_t}}, \tag{18e}$$

where $\epsilon = +1$ for $f = u, t$ and $\epsilon = -1$ for $f = b, \tau$. If we assume that $|\xi_f|$ is not very different from one for all fermions (as is the case in most models for the quark mass matrices), all parameters in (18a)–(18c) are expected to be $\mathcal{O}(1)$. Then, if no accidental cancellations occur, the order of magnitude for the electric and weak electric dipole form factors will be given by the coefficients $k_j^{B,f}$ in (17). On the other hand, in the case of the c quark we have to deal with contributions proportional to α'^c and β'^c, which contain the flavor-changing parameters μ_{ct}, μ_{tc} and μ_{sb}. These contributions depend on the Ansatz chosen for the quark-mass matrix, and can be very important due to the large top-quark mass.

To estimate the order of magnitude of the CP-violating form factors in different 2HDM scenarios, we have numerically calculated the values of the coefficients $k_j^{B,f}$ for different values of q^2 and the Higgs masses. Our results are presented in Tables 1–4. For the b, c, and τ form factors we have chosen $(q^2)^{1/2} = 10$, 92, 170 and 500 GeV, corresponding to the approximate center-of-mass energies in B-meson factories, LEP1, LEP2 and future e^+e^- colliders, respectively. For the t quark we have taken $(q^2)^{1/2} =$

Table 3. Dominant coefficients for $F_E^{Z,\tau}(s)$ and $F_E^{\gamma,\tau}(s)$ for $m_H = 100$ and 200 GeV and different values of s. All values are in units of $e$cm

	$s^{1/2} = 10\,\mathrm{GeV}$	$s^{1/2} = m_Z$	$s^{1/2} = 170\,\mathrm{GeV}$	$s^{1/2} = 500\,\mathrm{GeV}$
		$m_H = 100\,\mathrm{GeV}$		
$d^{Z,\tau}$	1.6×10^{-22}	1.8×10^{-22}	2.9×10^{-22}	$(-0.5 + 0.9\mathrm{i}) \times 10^{-22}$
$a^{\gamma,\tau}$	$(-1.2 - 0.7\mathrm{i}) \times 10^{-23}$	$(-1.0 - 4.8\mathrm{i}) \times 10^{-24}$	$(0.4 - 2.7\mathrm{i}) \times 10^{-24}$	$(0.4 - 0.5\mathrm{i}) \times 10^{-24}$
		$m_H = 200\,\mathrm{GeV}$		
$d^{Z,\tau}$	0.9×10^{-22}	0.9×10^{-22}	1.1×10^{-22}	$(-0.2 + 1.1\mathrm{i}) \times 10^{-22}$
$a^{\gamma,\tau}$	$(-0.4 - 0.2\mathrm{i}) \times 10^{-23}$	$(-1.1 - 1.6\mathrm{i}) \times 10^{-24}$	$(0.4 - 1.2\mathrm{i}) \times 10^{-24}$	$(0.1 - 0.4\mathrm{i}) \times 10^{-25}$

Table 4. Dominant coefficients for $F_E^{Z,c}(s)$ and $F_E^{\gamma,c}(s)$ for $m_H = 100$ and 200 GeV and different values of s. All values are in units of $e$cm

	$s^{1/2} = 10\,\mathrm{GeV}$	$s^{1/2} = m_Z$	$s^{1/2} = 170\,\mathrm{GeV}$	$s^{1/2} = 500\,\mathrm{GeV}$
		$m_H = 100\,\mathrm{GeV}$		
${a'}^{Z,c}$	0.6×10^{-21}	0.6×10^{-21}	0.7×10^{-21}	$(0.2 + 1.0\mathrm{i}) \times 10^{-21}$
$d^{Z,c}$	-0.7×10^{-21}	-0.7×10^{-21}	-1.2×10^{-21}	$(2.0 - 3.5\mathrm{i}) \times 10^{-22}$
$a^{\gamma,c}$	$(5.9 + 2.8\mathrm{i}) \times 10^{-24}$	$(0.5 + 1.9\mathrm{i}) \times 10^{-24}$	$(-0.1 + 1.1\mathrm{i}) \times 10^{-24}$	$(-1.6 + 2.0\mathrm{i}) \times 10^{-25}$
${a'}^{\gamma,c}$	1.7×10^{-21}	1.8×10^{-21}	1.9×10^{-21}	$(.6 + 2.9\mathrm{i}) \times 10^{-21}$
$(b' + c')^{\gamma,c}$	$(5.3 + 1.0\mathrm{i}) \times 10^{-24}$	$(1.0 + 1.6\mathrm{i}) \times 10^{-24}$	$(0.6 + 0.9\mathrm{i}) \times 10^{-24}$	$(0.5 + 7.5\mathrm{i}) \times 10^{-25}$
		$m_H = 200\,\mathrm{GeV}$		
${a'}^{Z,c}$	4.4×10^{-22}	4.6×10^{-22}	4.9×10^{-22}	$(2.5 + 7.2\mathrm{i}) \times 10^{-22}$
$d^{Z,c}$	-3.7×10^{-22}	-3.8×10^{-22}	-4.4×10^{-22}	$(1.0 - 4.6\mathrm{i}) \times 10^{-22}$
$a^{\gamma,c}$	$(1.6 + 0.7\mathrm{i}) \times 10^{-24}$	$(4.3 + 6.4\mathrm{i}) \times 10^{-25}$	$(1.4 + 5.0\mathrm{i}) \times 10^{-25}$	$(-0.6 + 1.6\mathrm{i}) \times 10^{-25}$
${a'}^{\gamma,c}$	1.3×10^{-21}	1.3×10^{-21}	1.4×10^{-21}	$(0.7 + 2.1\mathrm{i}) \times 10^{-21}$
$(b' + c')^{\gamma,c}$	$(1.6 + 0.3\mathrm{i}) \times 10^{-24}$	$(0.5 + 0.5\mathrm{i}) \times 10^{-24}$	$(2.7 + 4.1\mathrm{i}) \times 10^{-25}$	$(-0.2 + 1.4\mathrm{i}) \times 10^{-25}$

Table 5. Expected order of magnitude for electric and weak electric form factors at $q^2 = m_Z^2$ for the τ lepton and the b and c quarks in 2HDM I/II and Cheng–Sher-like scenarios. Values are in units of $e$cm

	$F_E^{Z,\tau}(m_Z^2)$	$F_E^{\gamma,\tau}(m_Z^2)$	$F_E^{Z,b}(m_Z^2)$	$F_E^{\gamma,b}(m_Z^2)$	$F_E^{Z,c}(m_Z^2)$	$F_E^{\gamma,c}(m_Z^2)$
2HDM I/II	10^{-22}	10^{-24}	10^{-21}	10^{-23}	10^{-21}	10^{-24}
Cheng–Sher	10^{-22}	10^{-24}	10^{-20}–10^{-21}	10^{-20}	10^{-21}–$10^{-21}\|\lambda_{ct}\|^2$	$10^{-21}\|\lambda_{ct}\|^2$

500 and 1800 GeV, the latter corresponding to the high-energy $\bar{p}p$ collider at Fermilab. We have considered neutral and charged scalar-boson masses of 100 and 200 GeV. In general, the Feynman integrals are expected to be suppressed if the masses of the scalars involved increase, so that the sums in (17) should be dominated by the contribution from the lightest Higgs. Notice that in the limit where the neutral scalar sector is degenerate in mass, the contributions from the diagrams Fig. 1a, d and e to both the electric and weak electric dipole form factors vanish owing to the orthogonality of the neutral-scalar mixing matrix O.

The contributions of the different CP-violation sources in the general 2HDM can be easily read from (18). Let us come back to the classification presented in the introduction: the CP violation in charged and neutral flavor-conserving scalar exchange is contained in the imaginary part of the parameters ξ_f, whereas the CP violation in neutral flavor-changing scalar exchange is due to the imaginary part of $\mu_{ff'}$. The CP violation in the scalar mixing matrix arises from the mixing between the CP-odd scalar I and the CP-even fields H^0 and R, that is, the products $O_{H^0 j}O_{Ij}$ and $O_{Rj}O_{Ij}$ in (18). Finally, the δ_{CKM} CP-violating phase in the V matrix only appears together with tree-level flavor-changing parameters in (18e).

In Table 5, we quote the expected orders of magnitude of the electric and weak electric form factors at $q^2 = m_Z^2$ for the τ lepton and the b and c quarks, in both 2HDM I/II and Cheng–Sher-like scenarios. For the 2HDM I and II, which include discrete symmetries to prevent FCNC, some of the contributions in (17) vanish. As stated in the previous section, in this case $\mathrm{Im}\xi_f = \mu_{ff'} = 0$, hence $\beta^f = \alpha'^f = \beta'^f = 0$, and the whole effect arises from the CP violation in the Higgs-mixing matrix. On the contrary, in the quark-mixing scheme proposed by Cheng and Sher, all terms in (17) have to be considered if the λ_{ij} parameters of (11) are complex numbers of order one. For the case of the b and c quarks, some of the terms that vanish in the 2HDM I and II have relatively large coefficients, proportional to the square of the top-quark mass. This shows up in the value of the corresponding electric dipole form factors, where the predictions in the Cheng–Sher scheme are about three orders of magnitude higher than in the 2HDM I/II, as can be seen in Table 5. In the case of the weak electric dipole form factors the effect is hidden due to the presence of other important contributions proportional to m_Z^2.

For the top quark the CP-violating form factors in the general 2HDM are dominated by the contributions of diagrams a and d in Fig. 1. These arise from flavor-conserving neutral Higgs exchange, and do not vanish in general in 2HDM I and II. As stated in the previous section, $F_E^{Z,t}$ and $F_E^{\gamma,t}$ have been analyzed previously for these models in [4]. The values for $F_E^{Z,t}(q^2)$ arising from (13a) and (13d) and the equivalent expressions for $F_E^{\gamma,t}(q^2)$ are in agreement with the results obtained in that paper. In addition, the CP-violating form factors for the top quark have been analyzed in a more general context in [15]. There, the authors consider a Cheng–Sher-like Ansatz for the Yukawa couplings, and assume that there is no mixing between the CP-odd and CP-even scalar fields. Hence they only obtain the contributions proportional to $\mathrm{Im}\xi_t$ and $\mathrm{Im}(\xi_t^2)$ in (18a) and (18c). The order of magnitude found in [15] for $F_E^{Z,t}$ and $F_E^{\gamma,t}$ for specific values of the Higgs parameters and CP-violating phases (namely about $10^{-20}\,e$cm for a center-of-mass energy of 500 GeV) is consistent with our results in Table 1.

The weak electric dipole moments of the τ lepton and b quark have been also calculated recently within the minimal supersymmetric standard model (MSSM) [5]. The results are similar to those obtained in our general 2HDM scheme: $|\mathrm{Re}[F_E^{Z,\tau}(m_Z^2)]| \lesssim 0.3(12) \times 10^{-21}\,e$cm, $|\mathrm{Re}[F_E^{Z,b}(m_Z^2)]| \lesssim 1.4(35) \times 10^{-21}\,e$cm. In [5] the authors also quote the values obtained for the top quark form factors at $(q^2)^{1/2} = 500$ GeV, which yield approximately $|F_E^{Z,t}| \simeq |F_E^{\gamma,t}| \simeq 10^{-19}\,e$cm. Taking the corresponding coefficients from Table 1, this order of magnitude agrees with our results. Notice that in our analysis we have not considered possible corrections arising from the running of the quark masses (we have used the running masses m_q in the $\overline{\mathrm{MS}}$ scheme, with $\mu = m_q$). In any case, these effects would not modify the quoted orders of magnitude for the values in Tables 1–5.

4 Conclusions

We compute the CP-violating electric and weak electric dipole form factors for heavy fermions in the framework of a completely general 2HDM. In spite of being one of the simplest extensions of the SM, this model contains interesting new features, such as the presence of various sources of CP violation beyond the standard CKM mechanism. The CP-violating dipole form factors are vanishingly small in the SM; thus they are good candidates to provide observable signals of new physics. In the 2HDM, at one loop, they are found to be finite and gauge invariant quantities, even when the involved γ or Z bosons are off-shell.

The effect of the different sources of CP violation on the form factors for the c, b and t quarks and the τ lepton is shown in (18). In particular, it is seen that some of the contributions vanish for the so-called 2HDM I and II, which include discrete symmetries to eliminate undesired FCNC. In these models the only remaining terms are those involving the mixing between CP-even and CP-odd neutral scalars. In the general case, however, these symmetries may be not present. If no accidental cancellations occur, this would imply an enhancement of three orders of magnitude in the electric dipole form factor of the b quark with respect to the prediction of 2HDM I and II for energies in the GeV to TeV range. On the other hand, we find that in the case of the c quark the electric dipole form factor is strongly dependent on the presence of c–t flavor-changing effects. Assuming an up-quark mass matrix of the type proposed by Cheng and Sher, with $\lambda_{uc} \sim \mathcal{O}(1)$, the values for $F_E^{\gamma,c}$ for energies from 10 to 500 GeV can be two to four orders of magnitude larger than those obtained in 2HDM I or II. We conclude from this analysis that the study of CP-violating dipole form factors can be useful to get information on the flavor mixing, and that it offers an interesting possibility to distinguish between these and other possible 2HDM scenarios.

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Appendix A: Feynman integrals

The integrals $I^{(\mathrm{i})}$ introduced in (13) are defined as follows:

$$\begin{aligned}
&\mathrm{Re}[I^{(\mathrm{I})}(m_q, m_{q'}, m_\phi, s)] \\
&\quad = \frac{1}{16\pi^2}\mathrm{P.V.}\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \frac{x}{f_1(x,y)}, \qquad \text{(A1)}\\
&f_1(x,y) = m_\phi^2(1-x-y) + (m_{q'}^2 - m_q^2)(x+y) \\
&\qquad\qquad + m_q^2(x+y)^2 - sxy,
\end{aligned}$$

$$\mathrm{Im}[I^{(\mathrm{I})}(m_q, m_{q'}, m_\phi, s)]$$

$$= \frac{1}{16\pi s}\frac{\beta_{q'}}{\beta_q^2}\left\{1+\frac{1}{\beta_q\beta_{q'}}\left(\frac{\beta_q^2-\beta_{q'}{}^2}{4}-\frac{m_\phi^2}{s}\right)\right.$$
$$\left.\times\log\left(\frac{s(\beta_q+\beta_{q'})^2+4m_\phi^2}{s(\beta_q-\beta_{q'})^2+4m_\phi^2}\right)\right\}\Theta(s-4m_{q'}^2), \quad \text{(A2)}$$

$$\mathrm{Re}[I^{(\mathrm{II})}(m_q,m_{q'},m_\phi,s)]$$
$$= \frac{1}{16\pi^2}\mathrm{P.V.}\int_0^1 \mathrm{d}x\int_0^{1-x}\mathrm{d}y\frac{1-2x}{f_2(x,y)}, \quad \text{(A3)}$$
$$f_2(x,y) = m_{q'}^2(1-x-y)+(m_\phi^2-m_q^2)(x+y)$$
$$+m_q^2(x+y)^2-sxy,$$

$$\mathrm{Im}[I^{(\mathrm{II})}(m_q,m_{q'},m_\phi,s)]$$
$$= -\frac{1}{8\pi s}\frac{\beta_\phi}{\beta_q^2}\left\{1-\frac{1}{\beta_q\beta_\phi}\left(\frac{\beta_q^2+\beta_\phi^2}{4}+\frac{m_{q'}^2}{s}\right)\right.$$
$$\left.\times\log\left(\frac{s(\beta_q+\beta_\phi)^2+4m_{q'}^2}{s(\beta_q-\beta_\phi)^2+4m_{q'}^2}\right)\right\}\Theta(s-4m_\phi^2), \quad \text{(A4)}$$

$$\mathrm{Re}[I^{(\mathrm{III})}(m_q,m_\phi,s)]$$
$$= -\frac{1}{16\pi^2}\mathrm{P.V.}\int_0^1 \mathrm{d}x\int_0^{1-x}\mathrm{d}y\frac{y}{f_3(x,y)}, \quad \text{(A5)}$$
$$f_3(x,y) = m_q^2(1-x-y)^2+m_\phi^2x+m_Z^2y-sxy,$$

$$\mathrm{Im}[I^{(\mathrm{III})}(m_q,m_\phi,s)]$$
$$= -\frac{1}{16\pi s}\left\{\frac{b_Z}{\beta_q^2}+\frac{1}{2\beta_q}\left(\frac{c-\beta_q^2}{\beta_q^2}+\frac{m_Z^2-m_\phi^2}{s}\right)\right.$$
$$\left.\times\log\left|\frac{c-b_Z\beta_q}{c+b_Z\beta_q}\right|\right\}\Theta[s-(m_Z+m_\phi)^2], \quad \text{(A6)}$$

$$\mathrm{Re}[I^{(\mathrm{IV})}(m_q,m_{q'},m_\phi,m_{\phi'},s)] \quad \text{(A7)}$$
$$= -\frac{1}{16\pi^2}\mathrm{P.V.}\int_0^1 \mathrm{d}x\int_0^{1-x}\mathrm{d}y\frac{(x-y)(1-x-y)}{f_4(x,y)},$$
$$f_4(x,y) = (m_{q'}^2-m_q^2)(1-x-y)+m_q^2(1-x-y)^2$$
$$+m_\phi^2x+m_{\phi'}^2y-sxy,$$

$$\mathrm{Im}[I^{(\mathrm{IV})}(m_q,m_{q'},m_\phi,m_{\phi'},s)]$$
$$= -\frac{1}{16\pi}\frac{(m_\phi-m_{\phi'})}{s^2}$$
$$\times\left\{\frac{2b_{\phi'}}{\beta_q^2}+\frac{d}{\beta_q}\log\left|\frac{d-b_{\phi'}\beta_q}{d+b_{\phi'}\beta_q}\right|\right\}$$
$$\times\Theta[s-(m_\phi+m_{\phi'})^2], \quad \text{(A8)}$$

with

$$\beta_a = (1-4m_a^2/s)^{1/2}, \quad a=q,q',\phi,$$
$$b_a = [(1-(m_\phi+m_a)^2/s]^{1/2}[1-(m_\phi-m_a)^2/s]^{1/2},$$
$$a = Z,\phi',$$
$$c = 1-(m_\phi^2+m_Z^2)/s,$$
$$d = 1-(m_\phi^2+m_{\phi'}^2)/s-2(m_q^2-m_{q'}^2)/s. \quad \text{(A9)}$$

In these expressions we assume $s > 4m_q^2$, which is valid for all the numerical estimations that are presented in this paper. The analytical results in (A2) and (A6) agree with those presented in [4] in the limit $\beta_{q'} = \beta_q$.

References

1. W. Bernreuther et al., Z. Phys. C **52**, 567 (1994); W. Bernreuther, O. Nachtmann, P. Overmann, Phys. Rev. D **48**, 78 (1993)
2. J. Bernabéu, G.A. González-Sprinberg, J. Vidal, Phys. Lett. B **326**, 168 (1994)
3. M. Acciarri et al., Phys. Lett. B **426**, 207 (1998); K. Ackerstaff et al., Z. Phys. C **74**, 403 (1997); D. Buskulic et al., Phys. Lett. B **346**, 371 (1995)
4. W. Bernreuther, T. Schröder, T.N. Pham, Phys. Lett. B **279**, 389 (1992)
5. W. Hollik, J.I. Illana, S. Rigolin, D. Stöckinger, Phys. Lett. B **425**, 322 (1998); W. Hollik, J.I. Illana, S. Rigolin, C. Schappacher, D. Stöckinger, Nucl. Phys. B **551**, 3 (1999)
6. C.A. Nelson, Phys. Rev. D **41**, 2805 (1990)
7. T. Barklow, contribution to TAU98, Santander, Spain, September 1998
8. S.L. Glashow, S. Weinberg, Phys. Rev. D **15**, 1958 (1977)
9. J. Liu, L. Wolfenstein, Nucl. Phys. B **289**, 1 (1987)
10. T.P. Cheng, M. Sher, Phys. Rev. D **35**, 3484 (1987)
11. A. Antaramian, L.J. Hall, A. Rašin, Phys. Rev. Lett. **69**, 1871 (1992); L.J. Hall, S. Weinberg, Phys. Rev. D **48**, 979 (1993)
12. Y.-L. Wu, L. Wolfenstein, Phys. Rev. Lett. **73**, 1762 (1994)
13. D. Atwood, L. Reina, A. Soni, Phys. Rev. D **55**, 3156 (1997)
14. T. Han, K. Whisnant, B.-L. Young, X. Zhang, Phys. Lett. B **385**, 311 (1996); S. Bar-Shalom, G. Eilam, A. Soni, Phys. Rev. Lett. **79**, 1217 (1997); Phys. Rev. D **57**, 2957 (1998); W.-S. Hou, G.-L. Lin, C.-Y. Ma, Phys. Rev. D **56**, 7434 (1997)
15. C.-H. Chang et al., hep-ph/9805408